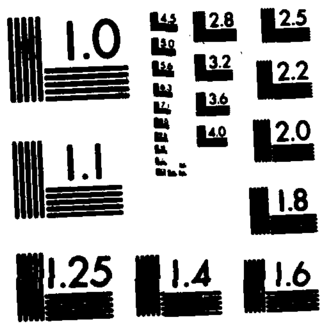
AD-A195 066 CIRCUIT DESIGN CRITERIA FOR STABLE LATERAL INHIBITION NEURAL NETWORKS..(U) MASSACHUSETTS INST OF TECH CAMBRIDGE MICROSYSTEMS RESEARCH CE.. J L WYATT ET AL. 1/1

UNCLASSIFIED MAR 88 VLSI-MEMO-88-439 N00014-80-C-0622 F/G 9/1 NL

END
FILMED
8



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(4)

MASSACHUSETTS INSTITUTE OF TECHNOLOGY VLSI PUBLICATIONS

VLSI Memo No. 88-439
March 1988

CIRCUIT DESIGN CRITERIA FOR STABLE LATERAL INHIBITION NEURAL NETWORKS

J. L. Wyatt, Jr. and D. L. Standley

Abstract

In the analog VLSI implementation of neural systems, it is sometimes convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. A serious problem of unwanted spontaneous oscillation often arises with these circuits and renders them unusable in practice. This paper reports a design approach that guarantees such a system will be stable, even though the values of designed elements in the resistive grid may be imprecise and the location and values of parasitic elements may be unknown. The method is based on a rigorous, somewhat novel mathematical analysis using Tellegen's theorem and the idea of Popov multipliers from control theory. It is thoroughly practical because the criteria are local in the sense that no overall analysis of the interconnected system is required, empirical in the sense that they involve only measurable frequency response data on the individual cells, and robust in the sense that unmodelled parasitic resistances and capacitances in the interconnect network cannot affect the analysis.

Microsystems Research Center Room 39-321
Massachusetts Institute of Technology
Cambridge Massachusetts 02139
Telephone (617) 253-8138

Accesion For		
NTIS	CRA&I	☑
DTIC	TAB	☐
Unannounced		☐
Justification		
By		
Distribution /		
Availability Codes		
Dist	Avail and / or Special	
A-1		

Acknowledgements

To appear in Proc. IEEE Int. Symp. on Circuits and Systems, Helsinki, Finland, June 1988. This work was supported in part by Defense Advanced Research Projects Agency through the Office of Naval Research under ARPA Order No. 3872, Contract numbers N00014-80-C-0622 and N00014-87-K-0825.

Author Information

Wyatt: Department of Electrical Engineering and Computer Science, MIT, Room 36-864, Cambridge, MA 02139, (617) 253-6718; Standley: Department of Electrical Engineering and Computer Science, MIT, Room 36-848, Cambridge, MA 02139, (617) 253-0454.

CIRCUIT DESIGN CRITERIA FOR STABLE LATERAL INHIBITION NEURAL NETWORKS

J.L. WYATT, Jr. and D.L. STANDLEY

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

In the analog VLSI implementation of neural systems, it is sometimes convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. A serious problem of unwanted spontaneous oscillation often arises with these circuits and renders them unusable in practice. This paper reports a design approach that guarantees such a system will be stable, even though the values of designed elements in the resistive grid may be imprecise and the location and values of parasitic elements may be unknown. The method is based on a rigorous, somewhat novel mathematical analysis using Tellegen's theorem and the idea of Popov multipliers from control theory. It is thoroughly practical because the criteria are local in the sense that no overall analysis of the interconnected system is required, empirical in the sense that they involve only measurable frequency response data on the individual cells, and robust in the sense that unmodelled parasitic resistances and capacitances in the interconnect network cannot affect the analysis.

I. INTRODUCTION

The term "lateral inhibition" first arose in neurophysiology to describe a common form of neural circuitry in which the output of each neuron in some population is used to inhibit the response of each of its neighbors. Perhaps the best understood example is the horizontal cell layer in the vertebrate retina, in which lateral inhibition simultaneously enhances intensity edges and acts as an automatic gain control to extend the dynamic range of the retina as a whole [1]. The principle has been used in the design of artificial neural system algorithms by Kohonen [2] and others and in the electronic design of neural chips by Carver Mead et. al. [3,4].

In the VLSI implementation of neural systems, it is convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. Linear resistors fabricated in, e.g., polysilicon, yield a very compact realization, and nonlinear resistive grids, made from MOS transistors, have been found useful for image segmentation. [4, 5]. Networks of this type can be divided into two classes: feedback systems and feedforward-only systems. In the feedforward case one set of amplifiers imposes signal voltages or currents on the grid and another set reads out the resulting response for subsequent processing, while the same amplifiers both "write" to the grid and "read" from it in a feedback arrangement. Feedforward networks of this type are inherently stable, but feedback networks need not be.

A practical example is one of Carver Mead's retina chips [3] that achieves edge enhancement by means of lateral inhibition through a resistive grid. Figure 1 shows a single cell in a continuous-time version of this chip. Note that the capacitor voltage is affected both by the local light intensity incident on that cell and by the capacitor voltages on neighboring cells of identical design. Any cell drives its neighbors, which drive both their distant neighbors and the original cell in turn. Thus the necessary ingredients for instability--active elements and signal feedback--are both present in this system, and in fact the continuous-time version oscillates so badly that the original design is scarcely usable in practice with the lateral inhibition paths enabled. [6] Such oscillations can

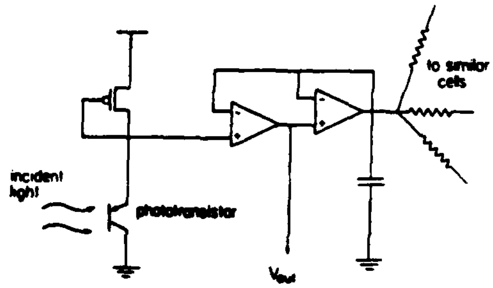


Figure 1. This photoreceptor and signal processor circuit, using two MOS transconductance amplifiers, realizes lateral inhibition by communicating with similar units through a resistive grid.

readily occur in any resistive grid circuit with active elements and feedback, even when each individual cell is quite stable. Analysis of the conditions of instability by straightforward methods appears hopeless, since the number of simultaneously active feedback loops is enormous.

This paper reports a practical design approach that rigorously guarantees such a system will be stable. The very simplest version of the idea is intuitively obvious: design each individual cell so that, although internally active, it acts like a passive system as seen from the resistive grid. In circuit theory language, the design goal here is that each cell's output impedance should be a *positive-real* [7] function. This is sometimes not too difficult in practice; we will show that the original network in Fig. 1 satisfies this condition in the absence of certain parasitic elements. More important, perhaps, it is a condition one can verify experimentally by frequency-response measurements.

It is physically apparent that a collection of cells that appear passive at their terminals will form a stable system when interconnected through a passive medium such as a resistive grid. The research contributions, reported here in summary form, are i) a demonstration that this passivity or positive-real condition is much stronger than we actually need and that weaker conditions, more easily achieved in practice, suffice to guarantee stability of the linear network model, and ii) an extension to the *nonlinear* domain that furthermore rules out large-scale oscillations under certain conditions.

II. FIRST-ORDER LINEAR ANALYSIS OF A SINGLE CELL

We begin with a linear analysis of an elementary model for the circuit in Fig. 1. For an initial approximation to the output admittance of the cell we simplify the topology (without loss of relevant information) and use a naive' model for the transconductance amplifiers, as shown in Fig. 2.

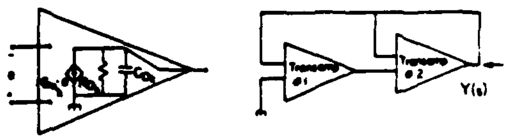


Figure 2. Simplified network topology and transconductance amplifier model for the circuit in Fig. 1. The capacitor in Fig. 1 has been absorbed into C_{o2}.

Straightforward calculations show that the output admittance is

$$Y(s) = [g_{m2}+R_{o2}^{-1}+s\,C_{o2}] + \frac{g_{m1}g_{m2}R_{o1}}{(1+s\,R_{o1}C_{o1})} \quad . \quad (1)$$

This is a positive-real, i.e., passive, admittance that could always be realized by a network of the form shown in Fig. 3, where

$R_1=(g_{m2}+R_{o1}^{-1})^{-1}$, $R_2=(g_{m1}g_{m2}R_{o1})^{-1}$, and $L = C_{o1}/g_{m1}g_{m2}$.

Although the original circuit contains no inductors, the realization has both capacitors and inductors and thus is capable of damped oscillations. Nonetheless, *if* the transamp model in Fig. 2 were perfectly accurate, no network created by interconnecting such cells through a resistive grid (with parasitic capacitances) could exhibit sustained oscillations since all the elements are passive. *For element values that may be typical in practice*, the model in Fig. 3 has a lightly damped resonance around 1 KHz with a Q = 10. This disturbingly high Q suggests that the cell will be highly sensitive to parasitic elements not captured by the simple models in Fig. 2. Our preliminary analysis of a

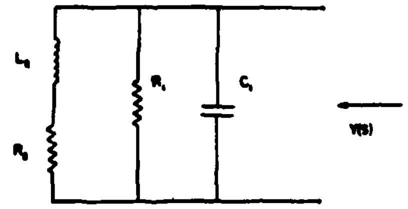


Figure 3. Passive network realization of the output admittance (eq. (1)) of the circuit in Fig. 2).

much more complex model extracted from a physical circuit layout created in Carver Mead's laboratory indicated that the output impedance will not be passive for all values of the transamp bias currents. But a definite explanation of the instability awaits a more careful circuit modelling effort and perhaps the design of an on-chip impedance measuring instrument.

III. STABILITY OF A LINEAR MODEL FOR THE NETWORK

Transistor parasitics and layout parasitics will cause the output admittance of the individual active cells to deviate from the form given in eq. (1) and Fig. 3, and any very accurate model will necessarily be quite high order. The following theorem shows what sort of deviations we can allow and still guarantee that the network is stable.

Terminology

The term *closed right half plane* refers to the set of complex numbers $s = \sigma + j\omega$ with $\sigma \geq 0$ and the term *closed third quadrant* refers to the set of complex numbers with $\sigma \leq 0$ and $\omega \leq 0$. A *natural frequency* is a complex frequency s_o such that, when all branch impedances and admittances are evaluated at s_o, there exists a nonzero solution for the complex branch voltages $\{V_k\}$ and currents $\{I_k\}$.

Theorem 1

Consider a linear network of arbitrary topology, consisting of any number of positive 2-terminal resistors and capacitors and of N lumped linear admittances $Y_n(s)$, n=1,2,...,N, having no poles or zeroes in the closed right half plane. Then the network is *stable*, in the sense that it has no natural frequency in the closed right half plane except perhaps at the origin, if at each frequency $\omega \geq 0$ there exists a phase angle $\theta(\omega)$ such that $0 \leq \theta(\omega) \leq 90°$ and $|\angle Y_n(j\omega) - \theta(j\omega)| < 90°$, n=1,2,...,N.

An equivalent statement of this last condition is that the Nyquist plot of each cell's output admittance for $\omega \geq 0$ never intersects the closed 3rd quadrant, and that no two cell's output admittance phase angles can ever differ by as much as 180°. If all the active cells are designed identically and fabricated on the same chip, their phase angles should track closely in practice and thus this second condition is a natural one.

Note that the above statement of the theorem does not rule out the possibility of an unusual instability arising from a repeated natural frequency at the origin. But a more careful argument, omitted here, shows that the only possible nonzero network solution at s=0 is the stable one in which capacitors in capacitor-only loops have nonzero d.c. voltages and all other branch voltages and currents vanish.

Proof of Theorem 1

Let s_o denote a natural frequency of the network and $\{V_k\}$ denote the complex branch currents at a corresponding solution. By Tellegen's theorem [8], or conservation of complex power, we have

$$\sum_{\text{resistances}} |V_k|^2/R_k + \sum_{\text{capacitances}} s_o C_k |V_k|^2 + \sum_{\substack{\text{cell}\\\text{output branches}}} Y_n(s_o)|V_n|^2 = 0. \tag{2}$$

Solutions of the form $s_o = j\omega \neq 0$ can be ruled out as follows. Note that for each $\omega>0$ all the cell admittance values $Y_n(j\omega)$ lie strictly above and to the right of a straight line through the origin of the complex plane making an angle of $\theta(\omega) - 90°$ with the real positive axis. The capacitance admittances $\{j\omega C_k\}$ and the resistor admittances $\{R_k^{-1}\}$ also lie above and to the right of this line. Thus no positive linear combination of these admittances can vanish as required by eq. (2).

To rule out solutions in the open right half plane, it is shown by a homotopy argument that the existence of such a solution implies the existence of a network satisfying the conditions of Thm. 1 and having natural frequencies of the form $s_o = j\omega \neq 0$ (already shown not to exist). Add a parallel conductance G to each element of the network, and call the parallel element pair a "composite element." Consider the locus of the natural frequencies as G is increased from zero to arbitrarily high values. Eventually they must all enter the open left half plane because all the composite elements become strictly passive at sufficiently high G values. Since the network started out with at least one open right half plane natural frequency, and the natural frequencies depend continuously on G, then there exists a G>0 such that the network has natural frequencies of the form $s_o = j\omega \neq 0$ ($s_o=0$ is ruled out by the strict passivity of all the composite elements here). It is easily verified that the collection of composite network elements satisfies the Thm. 1 conditions. Thus, open right half plane natural frequencies are ruled out. ∎

IV. STABILITY RESULT FOR NETWORKS WITH NONLINEAR RESISTORS AND CAPACITORS

The previous result for linear networks can afford some limited insight into the behavior of nonlinear networks. First the nonlinear equations are linearized about an equilibrium point and Theorem 1 is applied to the linear model. If the linearized model is stable, then the equilibrium point of the original nonlinear network is *locally stable*, i.e., the network will return to that equilibrium point *if* the initial condition is sufficiently near it. But the result in this section, in contrast, applies to the full nonlinear circuit model and allows one to conclude that in certain circumstances the network cannot oscillate *even if* the initial state is *arbitrarily far from* the equilibrium point.

Terminology

We say that a function $y=f(x)$ *lies in the sector* $[a,b]$ if $a\cdot x^2 \le x(fx) \le b\cdot x^2$. And we say that an impedance $Z(s)$ satisfies the *Popov criterion* if $(1 + rs)Z(s)$ *is positive real* [7,9,10] for some $r>0$. Note that this statement of the Popov criterion differs slightly from that given in standard references [9,10].

Theorem 2

Consider a network consisting of possibly nonlinear resistors and capacitors and cells with linear output impedances $Z_n(s)$, $n=1,2,\dots,N$. Suppose

i) the resistor curves are continuous functions $i_k = g_k(v_k)$ where g_k lies in the sector $[0,G_{max}]$ $G_{max}>0$, for all resistors,

ii) the capacitors are characterized by $i_k = C_k(v_k)\dot{v}_k$ where $0 \le C_k(v_k) \le C_{max}$ for all k and v_k, and

iii) the impedances $Z_n(s)$ all satisfy the Popov criterion for some common value of $r>0$. Then the network is stable in the sense that, for any initial condition,

$$\int_0^\infty \left[\sum_{\substack{\text{all resistors}\\\text{and capacitors}}} i_k^2(t) \right] dt < \infty . \tag{3}$$

Outline of Proof

By Tellegen's theorem, for any set of initial conditions and any time T>0,

$$\int_0^T \sum_{\text{resistors}} (v_k(t) + r\,\dot{v}_k(t))i_k(t)\,dt +$$

$$\int_0^T \sum_{\text{capacitors}} (v_k(t) + r\,\dot{v}_k(t))i_k(t)\,dt +$$

$$\int_0^T \sum_{\substack{\text{cell}\\\text{impedances}}} (v_k(t) + r\,\dot{v}_k(t))i_k(t)\,dt = 0. \tag{4}$$

For resistors, multiplying the sector inequality $v\,g(v) \le G_{max}v^2$ by $\dot{\phi} \ge 0$ yields $i^2 = ig(v) \le G_{max}iv$, and hence

$$G_{max}^{-1}\int_0^T i_k^2(t)\,dt \le \int_0^T i_k(t)v_k(t)\,dt =$$

$$\int_0^T i_k(t)[v_k(t) + r\,\dot{v}_k(t)]dt - r[\phi_k(v_k(t))-\phi_k(v_k(0))] \tag{5}$$

where

$$\phi_k(v) = \int_0^v g_k(v')dv' \geq 0 \tag{6}$$

is the *resistor co-content*. Using the inequality (6) in (5) yields

$$G_{max}^{-1}\int_0^T i_k^2(t)dt - r\phi_k(v_k(0)) \leq \int_0^T i_k(t)[v_k(t)+r\dot{v}_k(t)]dt. \tag{7}$$

For capacitors, integrating the inequality $i_k^2 = C_k^2(v_k)\dot{v}_k^2 \leq C_{max}C_k(v_k)\dot{v}_k^2$ yields

$$\frac{r}{C_{max}}\int_0^T i_k^2(t)dt \leq r\int_0^T C_k(v_k)\dot{v}_k^2(t)dt =$$

$$\int_0^T i_k(t)\,[v_k(t) + r\dot{v}_k(t)]dt - [E_k(q_k(T) - E_k(q_k(0)], \tag{8}$$

where

$$E_k(q) = \int_0^q v_k(q')dq' \geq 0 \tag{9}$$

is the capacitor energy. Using the inequality (9) in (8) yields

$$\frac{r}{C_{max}}\int_0^T i_k^2(t)dt - E_k(q_k(0) \leq \int_0^T i_k(t)[v_k(t)+r\dot{v}_k(t)]dt. \tag{10}$$

And for the cells, the assumption that $(1+rs)Z_n(s)$ is positive real implies that

$$\int_0^T i_n(t)\,[v_n(t) + r\dot{v}_n(t)]dt \geq -E_n(0), \tag{11}$$

where $E_n(0)$ is the "initial energy in the cell's output impedance" at t=0, a function of the initial conditions only. Substituting (7), (10) and (11) into (4) yields

$$G_{max}^{-1}\int_0^T \sum_{\text{resistors}} i_k^2(t)dt + \frac{r}{C_{max}}\int_0^T \sum_{\text{capacitors}} i_k^2(t)dt \leq$$

$$r\sum_{\text{resistors}}\phi_k(v_k(0)) + \sum_{\text{capacitors}} E_k(q_k(0)) + \sum_{\text{cells}} E_n(0)\ , \tag{12}$$

where the right hand side is a function only of the initial conditions. Thus (3) holds. $\blacksquare$

V. CONCLUDING REMARKS

The design criteria presented here are simple and practical, though at present their validity is restricted to linear models of the cells. There are several areas of further work to be pursued, one of which is an analysis of the differentiator cell that includes amplifier clipping effects. Others include the synthesis of a compensator for the differentiator cell, an extension of the nonlinear result to include impedance multipliers other than the Popov operator, and a waveform bounding analysis of the network which would guarantee adequate convergence after an allotted settling time.

ACKNOWLEDGEMENT

We sincerely thank Professor Carver Mead of Cal Tech for enthusiastically supporting this work and for making it possible for us to present an early report on it in this conference proceedings. This work was supported by Defense Advanced Research Projects Agency (DoD), through the Office of Naval Research under ARPA Order No. 3872, Contract No. N00014-80-C-0622 and Defense Advanced Research Projects Agency (DARPA) Contract No. N00014-87-K-0825.

REFERENCES

1. F.S. Werblin, "The Control of Sensitivity in the Retina," Scientific American, Vol. 228, no. 1, Jan. 1983, pp. 70-79.
2. T. Kohonen, Self-Organization and Associative Memory, (vol. 8 in the Springer Series in Information Sciences), Springer Verlag, New York, 1984.
3. M.A. Sivilotti, M.A. Mahowald, and C.A. Mead, "Real Time Visual Computations Using Analog CMOS Processing Arrays," Advanced Research in VLSI - Proceedings of the 1987 Standard Conference. P. Losleben, ed., MIT Press, 1987, pp. 295-312.
4. C.A. Mead, Analog VLSI and Neural Systems, Addison-Wesley, to appear in 1988.
5. J. Hutchinson, C. Koch, J. Luo and C. Mead, "Computing Motion Using Analog and Binary Resistive Networks," submitted to IEEE Transactions on Computers, August 1987.
6. M. Mahowald, personal communication.
7. B.D.O. Anderson, and S. Vongpanitlerd, Network Analysis and Synthesis - A Modern Systems Theory Approach, Prentice-Hall, Englewood Cliffs, NJ., 1973.
8. P. Penfield, Jr., R. Spence, and S. Duinker, Tellegen's Theorem and Electrical Networks, MIT Press, Cambridge, MA 1970.
9. M. Vidyasagar, Nonlinear Systems Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1970, pp. 211-217.
10. C. Desoer and M. Vidyasagar, Feedback Systems: Input-Output Properties, Academic Press, New York, 1975.

ATE

LMED

8